

Survey of A' Bhuidheanach Bheag

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1) **Introduction**

G&J Surveys has agreed a project with the Scottish Mountaineering Trust (SMT) to measure accurate heights for a range of Scottish mountains. The aim of the project is the resolution of anomalies that currently exist in several lists of the hills that are of interest to both the Scottish Mountaineering Club (SMC) and the wider hillwalking community. One such list is the Munros, hills in Scotland of height 3000 feet or more. The Ordnance Survey (OS) is the national authority responsible for the maintenance of Britain's geographical features and both the SMT and SMC feel it is in the interest of the hillwalking community that the heights of hills are officially verified by OS for inclusion on their mapping for the benefit of all.

A' Bhuidheanach Bheag (Hill Number 390, Section 5B, OS 1:50000 Map 42, OS 1:25000 Map OL51W, Grid Ref. NN660776) is marked on OS Maps with a trig pillar of height 936m and this has been recognised as the summit. However, the maps also show two separate contour rings exceeding 930m in height WSW and NE of the trig pillar at approximately 600m distance from the trig pillar. On this evidence there is the possibility that the hill's summit position may not be accurately identified.

The aim of this survey is to resolve this situation and obtain an accurate height for the summit using a survey grade Leica Viva GS 15 Professional GNSS (Global Navigation Satellite System) receiver and submit the data set collected to Ordnance Survey for verification. This will then lead to the height being included on Ordnance Survey mapping and enable the SMC and others to provide the officially recognised height in their future publications.

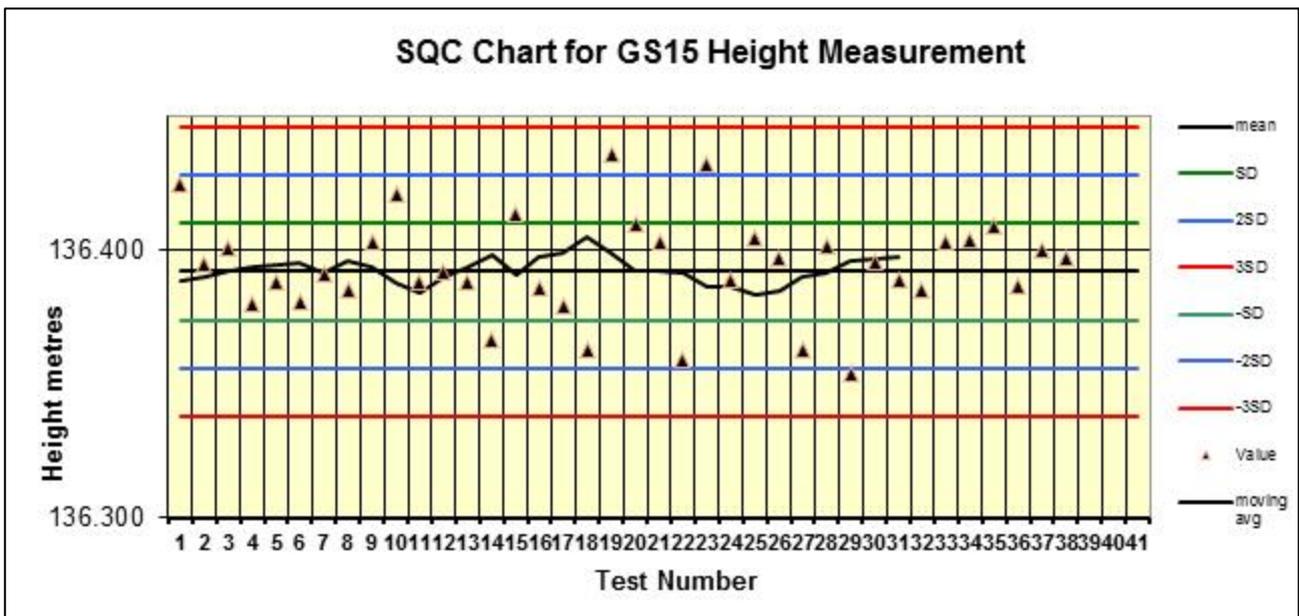
2) **Equipment used and Conditions for Survey**

Optical work and Line Surveys were carried out using a Leica NA730 Professional Automatic level (X30 telescopic system)/tripod system and a "1m" E-staff extendable to 5m.

Absolute heights were measured using a Leica Geosystems Viva GS15 Professional receiver. This instrument is dual-frequency and multi-channel, which means it can lock on to a maximum of 12 GPS and 8 GLONASS satellites as availability dictates, and receive two signals (at different frequencies) from each of these satellites. The latter feature reduces inaccuracies that result from atmospheric degradation of the satellite signals. As a stand-alone instrument, it is capable of giving position and height to an accuracy of about two metres and five metres respectively. Note that small hand-held GPS receivers used for general navigation can only receive up to 12 GPS satellites and each at a single frequency and therefore these instruments have a poorer positional accuracy of +/-5metres and a height accuracy of no better than +/-10 metres. Some recently produced hand held GPS Garmin receivers can also receive signals from GLONASS satellites which greatly improve the speed at which these units can achieve a satellite "fix". Despite the on-board features of the Viva GS15 receiver, there are still sources that create residual errors. To obtain accurate positions and heights, corrections were made to the GNSS (Global Navigation Satellite System) data via

imported RINEX data from the Ordnance Survey which were post-processed using Leica Geo Office 8.3 software.

The Leica NA730 level is routinely checked to make sure that the line of sight is correct when the instrument is set up horizontally; there is a standard surveying method to do this described in the users' manual for these instruments. We also regularly check the functioning of the Leica Viva GS15 GNSS receiver against Statistical Quality Control (SQC) charts generated for a marked position. The chart associated with height measurement is shown below. The mean height above ODN (Ordnance Datum Newlyn) for a fixed point (measured on 20 different occasions for 30mins of data collection at each time) was calculated to be 136.392m. Further height measurements have been made on separate occasions over a period of 18 months using the same process parameters. The last and penultimate measurements were carried out after and before the mountain survey described in this report. The results shown on the graph are all within a range of +/- three SD (Standard Deviation), in this case one SD is +/-0.018m and the moving average is within 1SD. This demonstrates that our Leica Viva GS15 GNSS receiver is giving consistently precise results within the expected errors for the measurements (all points are within a range of 0.07m of one another).



In addition, we check the instrument periodically by taking measurements on an Ordnance Survey Fundamental Bench Mark, processing the data and comparing it with the OS derived values. Height should agree within about 0.02-0.03m.

Checks were carried out on 27 September 2016 and 04 April 2017 at the Daresbury Fundamental Bench Mark and the results in the table below show excellent agreement between the Ordnance Survey measurement and our own.

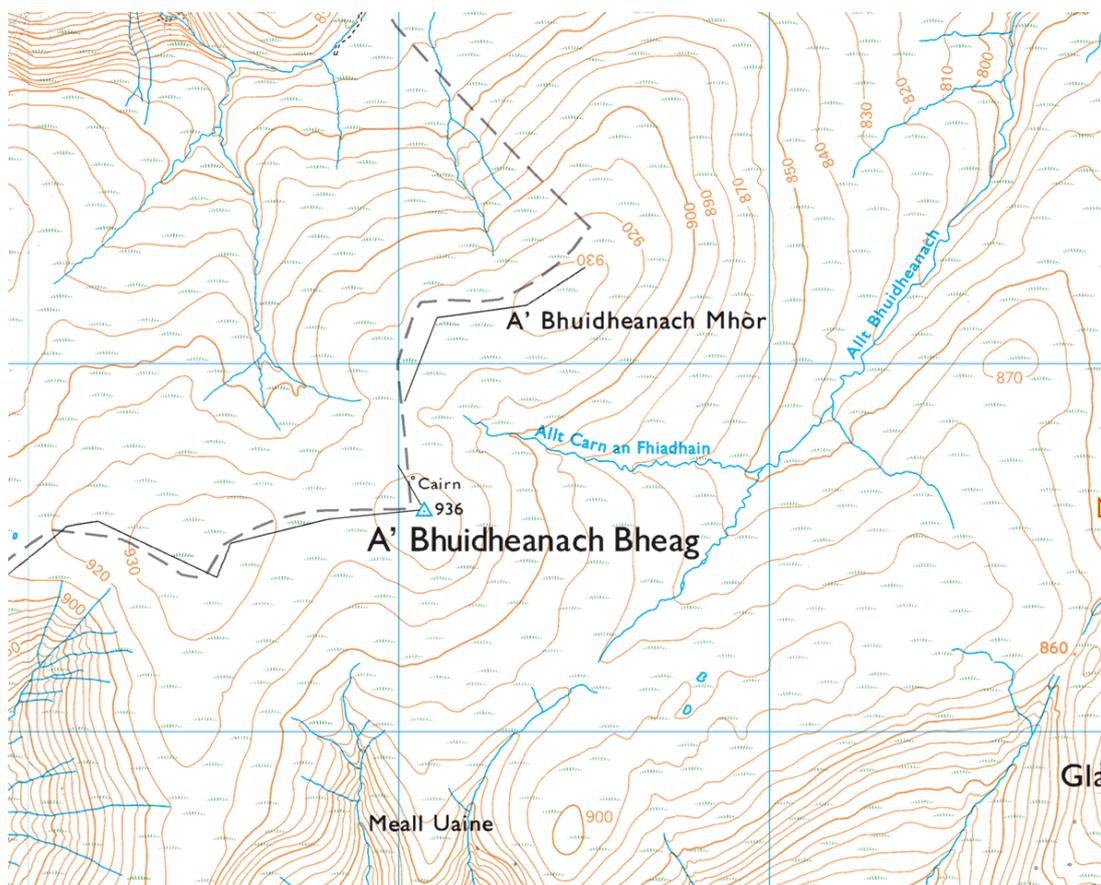
Processing	Date	Height(m)
OS measurement		73.24
JB/GVJ GeoOffice 8.3	27-09-2016	73.24
JB/GVJ GeoOffice 8.3	29-03-2016	73.23

Conditions were good for the survey which took place between 11.30 and 17.00hrs GMT. The weather was cold, 3 degrees Celsius, with a 15mph wind. Although the sky was cloudy, visibility was good. Snow had fallen earlier in the week which was quite deep in the cols but on the tops it was light and did not interfere with the survey.

3) Character of Hills

A' Bhuidheanach Bheag is classified as a Munro and lies on the East side of the A9 in the Drumochter Pass near Dalwhinnie. (To the NNE, about 5km distance, lies another Munro, Carn na Caim whose South Top was identified as reaching 3000 feet in a survey carried out in this programme with the SMT). The hills in this region are rounded and covered with thick heather and grass which makes ascent quite tedious unless a track can be found. Since Drumochter Pass exceeds 400m, the highest ground to the East only reaches about 500m above the valley bottom and hence the Munros here do not appear very high.

Access is most easily gained from layby 88 on the A9 from where a wide track leads up the hillside to a disused quarry at just under 900m. From there the broad summit ridge can be followed south to A' Bhuidheanach. Here the ridge turns SE to the NE top of A' Bhuidheanach Mhor and then SW to the summit of A' Bhuidheanach Bheag. (Often hillwalkers will combine the ascent of A' Bhuidheanach Bheag with Carn na Caim through a circular walk from the West). An extract of the Ordnance Survey (Crown Copyright) 1:25000 scale map shown below shows the features of the summit area. A line of old fence posts also marks out the route than can be followed to the three separate tops.



4) The Survey

The survey commenced at the area around the trig point. The Leica NA730 level was first used to identify the highest point. The Leica GS15 receiver was then set up over this point and GNSS data were collected in order to measure its absolute height.

While the GS15 was collecting data over the summit position, a line survey was carried out to the West Top. Our standard procedure using “triple wire levelling” was used. The staff was held vertically on the summit position and the Leica NA730 automatic level set up on a tripod in a convenient position in the direction towards the West Top. Once a set of readings had been taken (Backsight Reading), the staff was then moved to a position further towards the West Top, but the level was not moved apart from a rotation through “180 degrees” to take another set of readings (Foresight Readings). Each set comprised a reading of the central level line and the upper and lower stadia lines. The average of these three readings was then calculated and, provided this average was within 1mm of the central line reading, then the set was accepted and the line survey continued. The process of alternately moving the staff and level was repeated until the final reading was taken with the staff positioned at highest point on the West Top. The whole procedure was then reversed to survey from the West Top back to the summit by the trig point.

Next, the Leica GS15 receiver was set up over the highest point on the West Top and GNSS Data were collected there in order to measure its absolute height so that the difference between that and the summit by the trig point could be compared with the results from the Line Survey.

Finally, we returned to the summit by the trig point and set up the NA730 level so that it was horizontal with the summit position. The NE Top was then observed through the level and was clearly lower. A photograph was taken through the level so that a quantitative measurement of the height difference between the hill’s summit and the NE Top could be made.

4.1) Summit next to Trig Point

The area around the base of the trig point was quite flat and therefore the highest point was not obvious. The Leica NA730 level was set up on a tripod at a convenient position a few metres away from the trig point but in a position to be able to observe the trig point flush bracket. Systematic staff readings radiating away from the trig point were then taken until the highest point had been found. This position was measured to be 11.3 metres distant from the trig point and on a bearing of 240 degrees from it. A staff reading was also taken from the flush bracket so that our height measurement could be compared with that in the OS Database for trig points.

Staff reading at summit = 0.710m

Staff reading at flush bracket = 0.674m

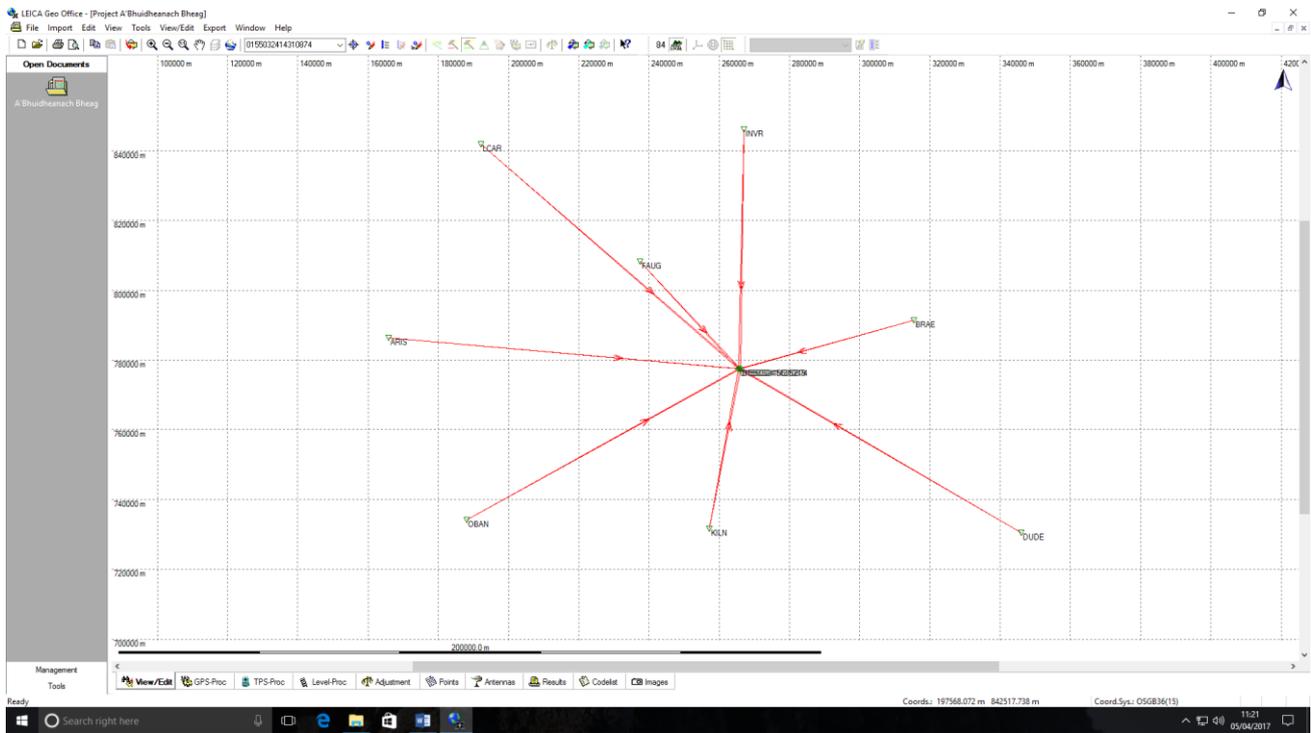
Flush bracket to base of trig point = 0.160m

Flush bracket is $0.710 - 0.674 = 0.036\text{m}$ higher than summit.

The Leica GS15 receiver was mounted on a 2.000 metre pole supported by a Leica “Quickset” tripod over the summit position (see photograph in Appendix 1). GNSS data were collected at the point for 2 hours with an epoch time of 15 seconds.

The data for the Leica Viva GS15 were processed using Leica GeoOffice 8.3 and we used the eight nearest base stations: (Fort Augustus – FAUG 42km, Killin – KILN 46km, Braemar - BRAE 51km, Inverness – INVR 69km, Oban – OBAN 89km, Dundee - DUDE 93km, and Arisaig – ARIS 100km. We used Broadcast Ephemeris data received by the GNSS receiver during the survey rather than Precise Ephemeris data, since we have found this makes little difference to the height results. The computed Tropospheric model was chosen for the calculations to suit the data collection times and the wide difference in height between the base stations and the summit of the mountain.

The distances and directions of the base stations from A’ Bhuidheanach Bheag are shown in the scaled diagram below. As far as is possible, the base stations are evenly distributed around the survey point and heights measured from each base station were within +/-0.06m of the mean result for the summit.



The results are shown in the Table below: -

	Easting	error(1SD)	Northing	error(1SD)	Height(m)	error(1SD)
Summit	266058.738	0.003	777594.049	0.002	936.029	0.008

The measured height for the Flush bracket on the trig point is $936.029 + 0.036 = 936.065\text{m}$. The OS Database gives the height of the Flush Bracket as 936.345m . The difference is 0.28m which is towards the upper end of the agreement between the results for comparisons we have made previously.

4.2) Line Survey between Summit and West Top

The line survey was carried out as described in Section 4 and the measurements taken are shown in Appendix 3. Although the distance between the two tops is 570m there is only 11m of drop between the two. Therefore, this means that the line survey could be completed in fewer measurements and hence more quickly. We chose the direct line along the old line of fence posts and despite the thicker snow between the two tops, staff and level setup positions were easily found. In accord with good practice we tried to balance Backsight and Foresight measurement distances as this eliminates any collimation error in the adjustment of the Leica NA730 level. The imbalances in these distances were small at 14m and 31m respectively for the two line surveys over a total distance of about 650m .

The average measured height difference between the two tops for the two line surveys was 0.621m with a closing error of 0.020m . This small closing error shows good agreement between the two line surveys.

The height of the West Top is $936.029 - 0.621 = 935.41\text{m}$

4.3) West Top

The highest point of the West Top was found on the last forward measurement of the line survey. At first glance the highest point appeared to be a small “mound” by the right-angled turn of the fence posts. However systematic staff measurements showed that ground to the North West was marginally higher and therefore the summit of the West Top. Subsequently the Leica GS15 was mounted on a 2.000 metre pole over this point and was supported with a Leica “Quickset” tripod. As the height of the West Top had already been found from the Line Survey, the GNSS data were collected to identify the high point location and to confirm the Line Survey measurement. Therefore only 1 hour of GNSS data were collected with an epoch time of 15 seconds. The computation in GeoOffice 8.3 was carried out using the same parameters as for the Summit calculation described in Section 4.1

The results for the GS15 measurements are shown in the table below: -

	Easting	error(1SD)	Northing	error(1SD)	Height(m)	error(1SD)
Summit	265506.715	0.002	777449.253	0.002	935.380	0.010

The measured height of 935.38m compares well with the height calculated from the line survey of 935.41m as is within the expected measurement uncertainty.

4.4) North East Top

The North East Top lies about 700m distant from the trig point. This Top was not visited during this survey. However, as described earlier in this report, optical measurements from ground near the trig point showed that the North East Top was too low to be a contender for the summit position of A' Bhuidheanach Bheag. Appendix 2 shows a photograph taken from the summit near the trig point to the North East Top through the Leica NA730 level with it set on a tripod to the same height as the summit.

From a print of the photograph it is possible to make a quantitative estimate of the height of the North East Top. The distance between the North East Top and the summit can be estimated to be 700m from the 1:25000 OS Map. This means that with a 100:1 stadia ratio built into the Leica NA730 level, the vertical distance between the upper and lower horizontal stadia lines represents a height of 7.0m. This distance has been measured on a photographic print to be 15cm and therefore we have a height calibration 0.47m per 1cm of photographic print. The highest point of the North East Top lies 5.3cm below the horizontal level line which represents 2.5m of height.

We can therefore estimate the height of the North East Top to be $936.0 - 2.5 = 933.5\text{m}$

5) Coordinate Recovery Analysis

In order to verify the accuracy and consistency of a GNSS dataset, Ordnance Survey recommends a procedure called Coordinate Recovery Analysis. Instead of processing the data with reference to all the nearest OS Base Stations under approximately 100km distance, as used in this survey, the data is first processed with reference to only the nearest Base Station. The data is then reprocessed with the survey point taken as a Reference Point and all the remaining Base stations taken as survey points. These measured values for the OS Base Stations can then be compared directly with the actual OS values for Position and Height. (This has been carried out via an Excel Spreadsheet supplied to us by OS).

Although the spreadsheet calculates a number of different parameters, two important ones are presented in the tables below. “Height Difference U metres” is the vertical height difference between the height of the Base Station as measured in this survey compared with the actual OS value. “Separation D_{ij} metres” is the distance in 3-d space between the measured and actual OS values for each Base Station.

The results for the survey are presented below.

Base Station	Code	Distance to Survey Point km.	Height Difference U metres	Separation D_{ij} metres
Fort Augustus	FAUG	42		
Killin	KILN	46	0.0087	0.0129
Braemar	BRAE	51	0.0123	0.0140
Inverness	INVR	69	0.0095	0.0095
Oban	OBAN	89	0.0012	0.0168
Dundee	DUDE	93	0.0840	0.0849
Lochcarron	LCAR	98	0.0068	0.0116
Arisaig	ARIS	101	-0.0125	0.0181
Buckie	BUCK	117	-0.0044	0.0178

The results for A’ Bhuidheanach Bheag show a consistent dataset as all measured OS Base stations are within 0.02m distance except for Dundee (DUDE) which has a separation of 0.085m. However, the OS specify that Coordinate Recovery is acceptable if the Separation is less than 0.1m.

6) Summary of GNSS parameters and Discussion of Errors

The summary of the conditions used for the GS15 GNSS measurements are shown in the table below: -

Variable	Summit	West Top
Data collection summit (min)	139	62
Number of Base Stations used in Processing for all points	8	8
Epoch Time (sec)	15	15
Tropospheric Model	Computed	Computed
Geoid Model	OSGM15	OSGM15
Cut off Angle (degs)	15	15

For a 1 hour and a 2 hour GNSS dataset, previous work has shown the measurement uncertainty in height to be +/-0.06m and +/-0.05m respectively, as defined by 3 times the standard deviation. The associated uncertainty in height in determining the correct position for tops we estimate to be +/-0.02m. Therefore, the overall uncertainty in any absolute height measurement is +/-0.06m.

The line survey closing error was $\pm 0.01\text{m}$ from the mean line survey result. Since the uncertainty in height for the location of each summit was estimated to be $\pm 0.02\text{m}$, that for the GNSS measurement of the summit at $\pm 0.05\text{m}$, the uncertainty in the absolute height measurement for the West Top by this method is $\pm 0.06\text{m}$.

7) Summary and Conclusions

The **summit** of **A' Bhuidheanach Bheag** is at grid reference * NN 66060 77585 and is on unfeatured ground 11m from the trig point and on a bearing of 240 degrees from it. **Its height is 936.0 \pm 0.06m.**

The highest point of the West Top is an embedded rock at Grid Reference * NN 65508 77440. **Its height is 935.4 \pm 0.06m.**

* Grid references given in Summary and Conclusions are for users of Garmin GNSS receivers

The results have been accepted by Ordnance Survey and forwarded to OS Cartography for the relevant map changes.

Acknowledgements

Many people contributed to the success of this survey.

We would especially like to thank the Scottish Mountaineering Trust for generously supporting the work and Rab Anderson and Andy Nisbet of the Scottish Mountaineering Club for their guidance and encouragement.

We also wish to thank Mark Greaves of the Ordnance Survey, who accepted the data and forwarded the results to OS Cartography for map changes. We also thank Mark for his support and advice that has helped us carry out our mountain heighting work over the past seven years.

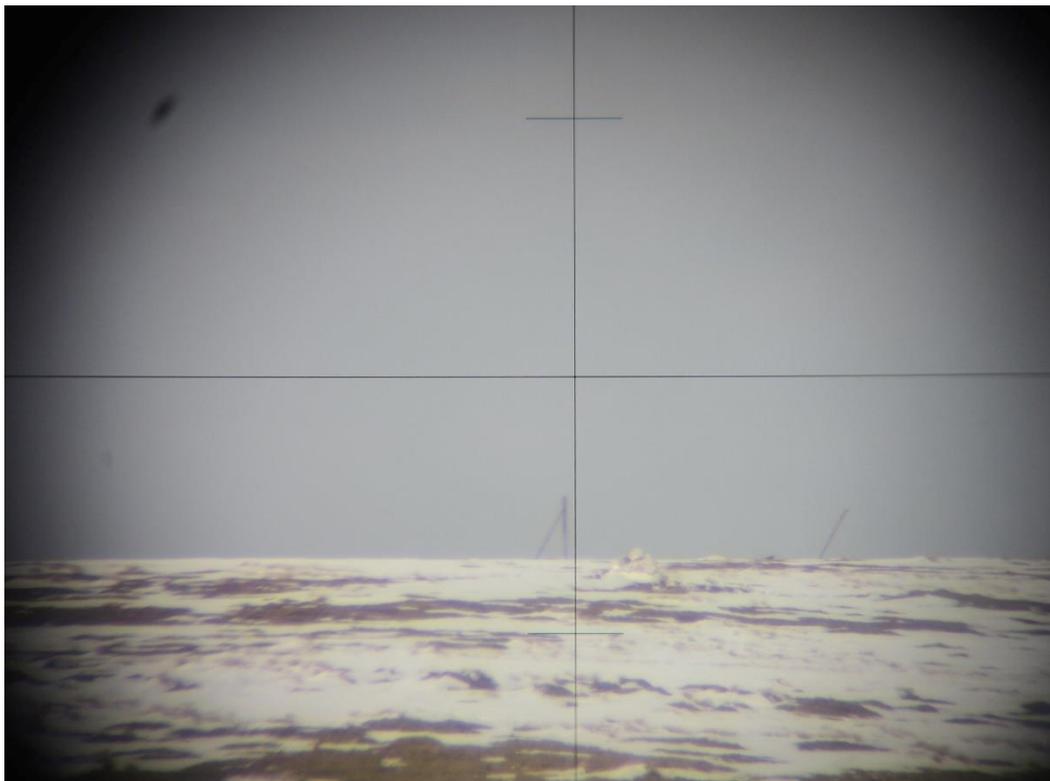
John Barnard and Graham Jackson, 05 April 2017.

Appendix 1



Leica Viva GS 15 set up on summit of A' Bhuidheanach Bheag

Appendix 2



NE Top of A' Bhuidheanach Bheag photographed through the Leica NA730 Level

Appendix 3

Title:- A'Bhuidheanach Bheag

Instrument:- Leica NA730

24-Mar-17

Point Number	Horizontal Line		Lower Stadia Line		Upper Stadia Line		Mean BS	Mean FS	Ht Difference	BS Distance	FS Distance
	Backsight R metres	Foresight F metres	Backsight R metres	Foresight F metres	Backsight R metres	Foresight F metres					
Summit by Trig Point to West Top (CC Staff, JB Level and DM Data recording)											
1	0.710	2.612		2.365		2.858	0.710	2.612		15.000	49.300
2	0.175	3.440	0.116	3.190	0.235	3.688	0.175	3.439		11.900	49.800
3	0.215	3.761	0.161	3.481	0.269	4.044	0.215	3.762		10.800	56.300
4	0.257	3.072	0.192	2.769	0.323	3.375	0.257	3.072		13.100	60.600
5	2.474	0.525	2.076	0.360	2.871	0.690	2.474	0.525		79.500	33.000
6	4.089	0.267	3.791	0.217	4.386	0.317	4.089	0.267		59.500	10.000
7	4.323	0.693	4.010	0.461	4.638	0.925	4.324	0.693		62.800	46.400
8	2.118	0.602	1.756	0.434	2.480	0.773	2.118	0.603		72.400	33.900
						Sum =	14.362	14.973	-0.611	325.000	339.300
West Top to Summit by Trig Point (CC Staff, JB Level and DM Data recording)											
1	0.602	2.115	0.434	1.755	0.773	2.479	0.603	2.116		33.900	72.400
2	0.689	4.321	0.460	4.004	0.916	4.640	0.688	4.322		45.600	63.600
3	0.373	4.283	0.327	3.925	0.418	4.642	0.373	4.283		9.100	71.700
4	0.425	2.113	0.220	1.869	0.630	2.359	0.425	2.114		41.000	49.000
5	3.418	0.411	2.990	0.344	3.849	0.476	3.419	0.410		85.900	13.200
6	4.215	0.200	3.947	0.139	4.485	0.262	4.216	0.200		53.800	12.300
7	4.595	0.242	4.255		4.935		4.595	0.242		68.000	24.500
						Sum =	14.319	13.688	0.631	337.300	306.700